A 2D Vortex Panel Method approach for modelling unsteady airfoil dynamics

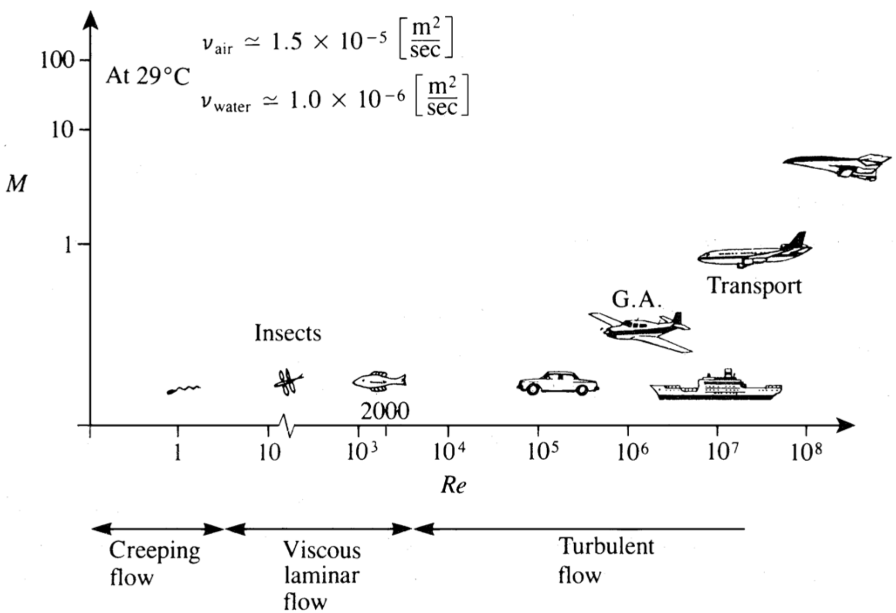
Vedang Joshi
Department of Engineering
King's College London

Introduction

- Emergence of fish schools by studying the role of vortex shedding behind fish, using techniques from unsteady fluid dynamics.

Fish schools provide hydrodynamic benefits to individuals through flow-mediated interactions[1].

Fish schools achieve improved propulsive performance by harvesting energy from vortex wakes[2] enhancing thrust production[3] or by reducing drag[4].



$$M \ll 1, \qquad \frac{M^2}{Fr^2} \ll 1, \qquad \frac{M^2}{Re} \ll 1$$

[1] Weihs, D. (1973)
[2] Liao et. al (2003)
[3] Boschitsch et. al (2014)
[4] Maertens et. al (2017)
[5] JSTOR
[6] Katz, J. and Plotkin, A. (2001)

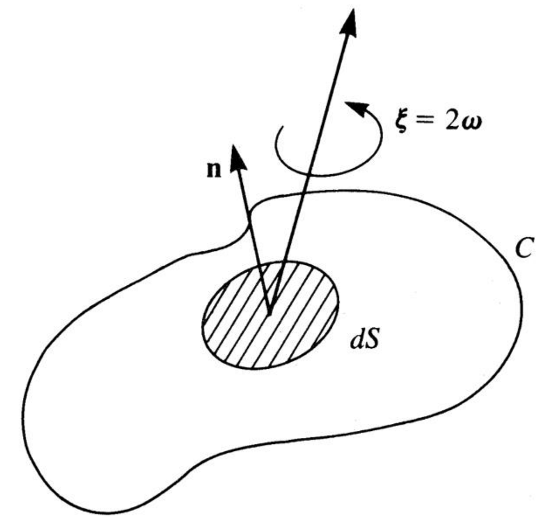
Rapid intro to Inviscid, Incompressible Flow

The vorticity is twice the angular velocity

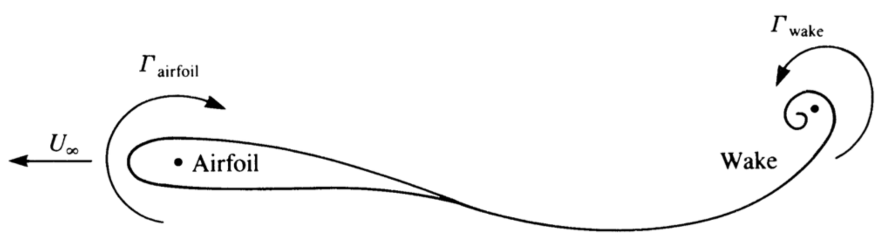
$$\zeta \equiv 2\omega = \nabla \times \mathbf{q}$$

We have an open surface S, and closed curve C

$$\int_S \nabla \times \mathbf{q} \cdot \mathbf{n}\, dS = \int_S \zeta \cdot \mathbf{n}\, dS = \oint_C \mathbf{q} \cdot d\mathbf{l}$$



Kelvin's theorem: Rate of change of circulation around closed curve with the same fluid elements is zero



$$\frac{D\Gamma}{Dt} = \frac{\Gamma_{\text{airfoil}} + \Gamma_{\text{wake}}}{\Delta t} = 0$$

Biot-Savart Law: Determine the velocity field as a result of a known vorticity distribution

$$\mathbf{q} = \nabla \times \mathbf{B}$$

$$\zeta = \nabla \times \mathbf{q} = \nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B}$$

$$\mathbf{B} = \frac{1}{4\pi} \int_V \frac{\zeta}{|\mathbf{r}_0 - \mathbf{r}_1|} dV \quad \mathbf{q} = \frac{1}{4\pi} \int_V \nabla \times \frac{\zeta}{|\mathbf{r}_0 - \mathbf{r}_1|} dV$$

Take a small cross sectional area dS, normal to the vorticity, with direction **dl** on the filament:

$$\nabla \times \frac{\zeta}{|\mathbf{r}_0 - \mathbf{r}_1|} dV = \nabla \times \Gamma \frac{d\mathbf{l}}{|\mathbf{r}_0 - \mathbf{r}_1|} = \Gamma \frac{d\mathbf{l} \times (\mathbf{r}_0 - \mathbf{r}_1)}{|\mathbf{r}_0 - \mathbf{r}_1|^3}$$

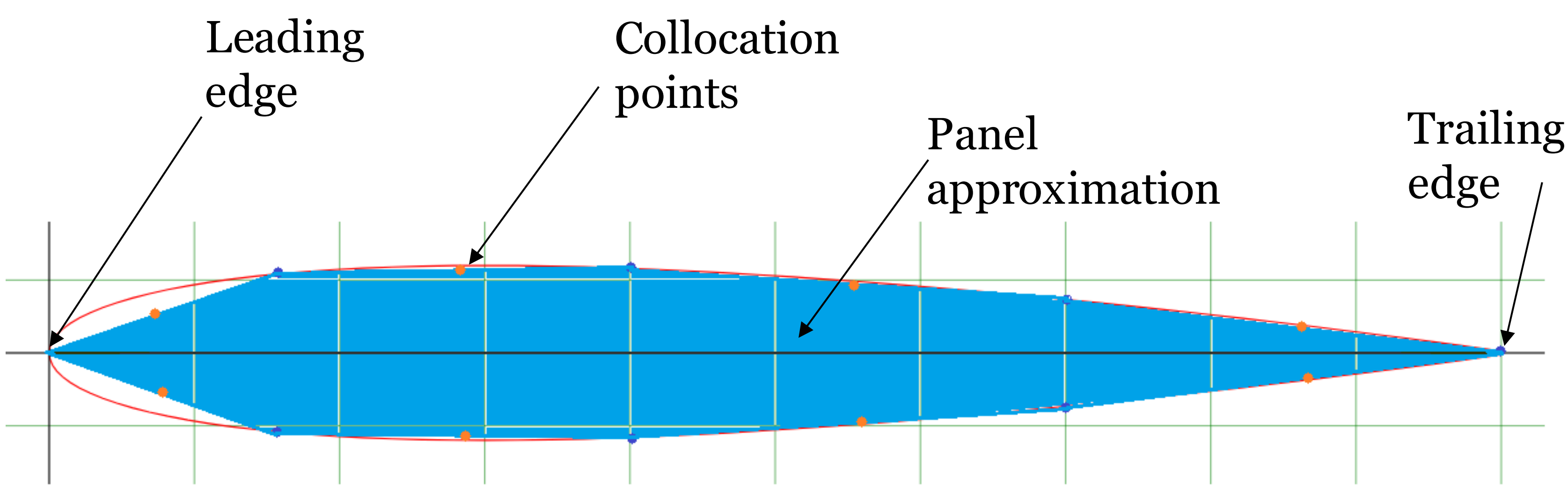
$$\mathbf{q} = \frac{\Gamma}{4\pi} \int \frac{d\mathbf{l} \times (\mathbf{r}_0 - \mathbf{r}_1)}{|\mathbf{r}_0 - \mathbf{r}_1|^3}$$

[1] Katz, J. and Plotkin, A. (2001)

Panel Methods

- Technique for solving incompressible potential flow over 2D and 3D geometries
- In 2D, the airfoil surface is divided into piecewise straight-line segments/panels/boundary elements point vortex singularities of strength γ, are placed on each panel
- Greater number of panels, the more accurate the solution.
- We apply the boundary condition at the control point, treating the airfoil surface as a streamline. Velocity would be tangential to the surface, and no fluid can penetrate the surface

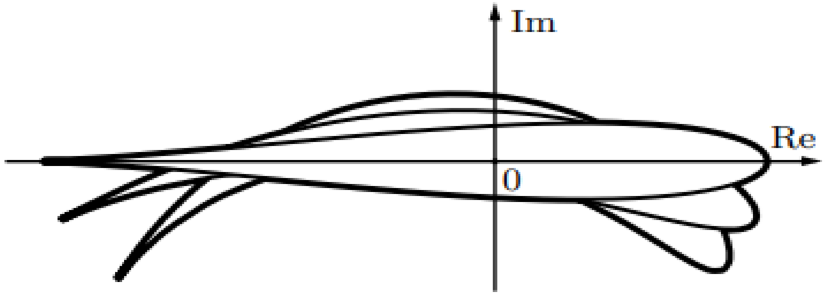
The net effect of viscosity on a wing is captured by the Kutta condition, which requires that the flow leaves the sharp trailing edge smoothly, with no infinite velocities or flow separation.



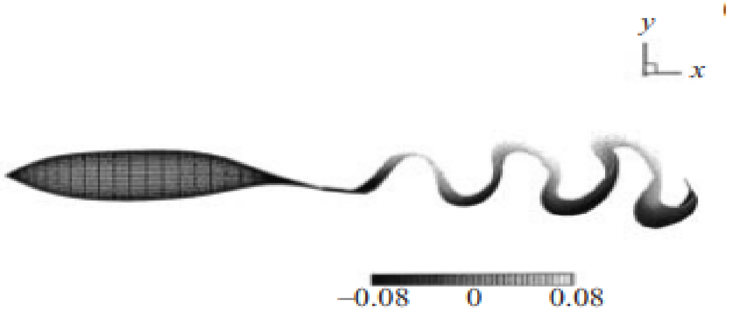
[1] Katz, J. and Plotkin, A. (2001)

Related Work

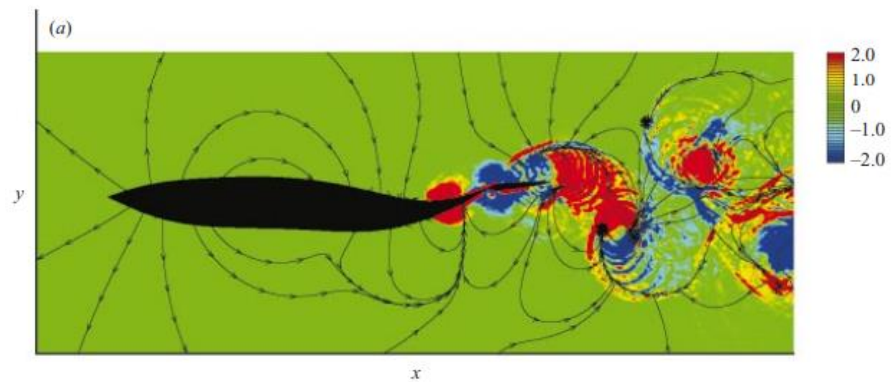
- Allows for computation of arbitrary profile deformation that cannot easily be defined by conformal transformations[1].
- Studied for individuals by developing a fish-like profile, imposing a deformation parameter such that the profile bends while maintaining camber length and area[1].



(b) Body frame $\mathcal{F}_B$



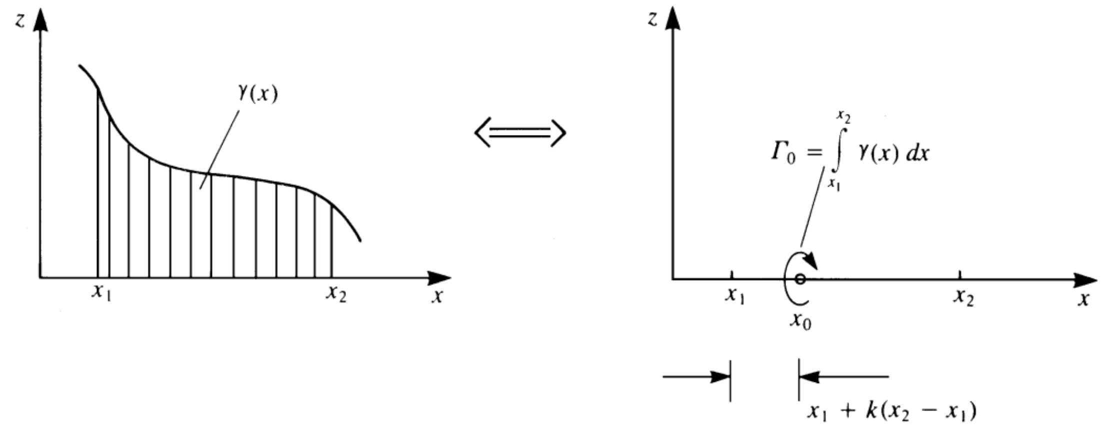
- Panel methods are implemented for numerical simulations on the giant Danio, with the caudal fin having chordwise sections of NACA 0016 shape[2].



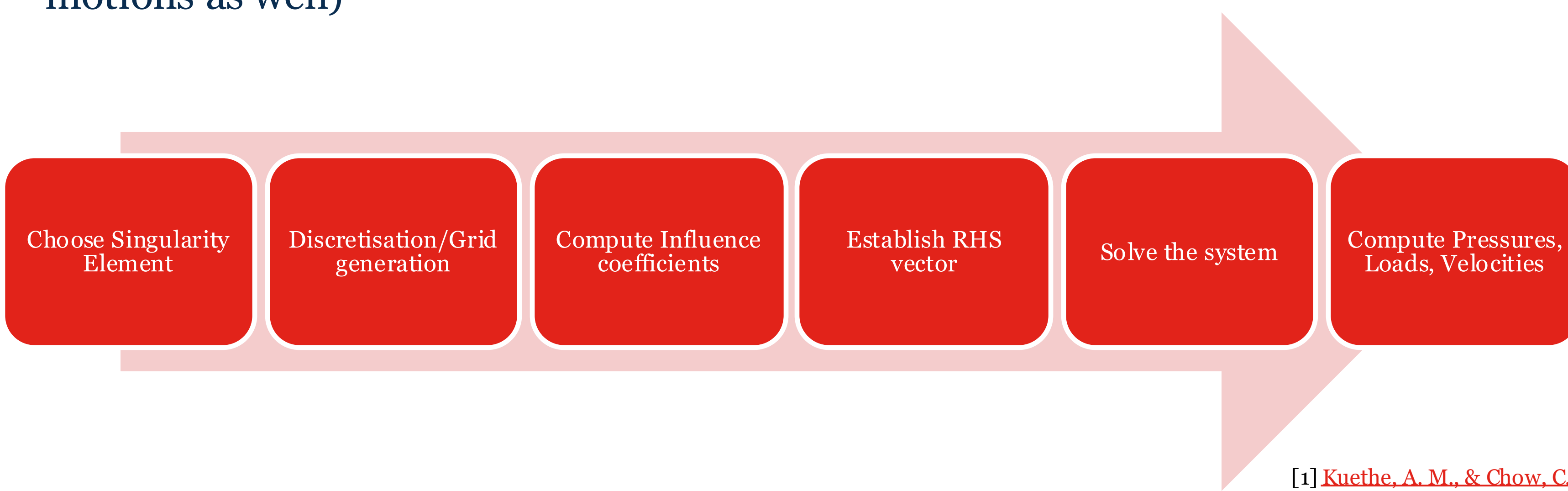
[1] Xu, Y., & Mohseni, K. (2015)
[2] Zhu et. al (2002)

Vortex Panel Method

- Point singularity solutions
- Able to discretise γ (x) into finite segments
- Influence coefficient calculations calculated using Kuethe and Chow[1]
- The following slides follow Katz and Plotkin's formulation



- Steady and Unsteady vortex panel methods created for the NACA0012 airfoil undergoing sudden forward motion (may be extended to pitching and heaving motions as well)



[1] Kuethe, A. M., & Chow, C. Y. (1997)
[2] Katz, J. and Plotkin, A. (2001)

Modifications for the unsteady state: Sudden Forward Motion

Kinematics subroutine[1]

- $X_0 = -Q_\infty t \qquad \dot{X}_0 = -Q_\infty$
- $Z = 0 \qquad \dot{Z}_0 = 0$
- $\theta = \alpha \qquad \dot{\theta}_0 = 0$

$$\begin{pmatrix} X \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta(t) & \sin\theta(t) \\ -\sin\theta(t) & \cos\theta(t) \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} X_0 \\ Z_0 \end{pmatrix}$$

RHS vector modification[1]

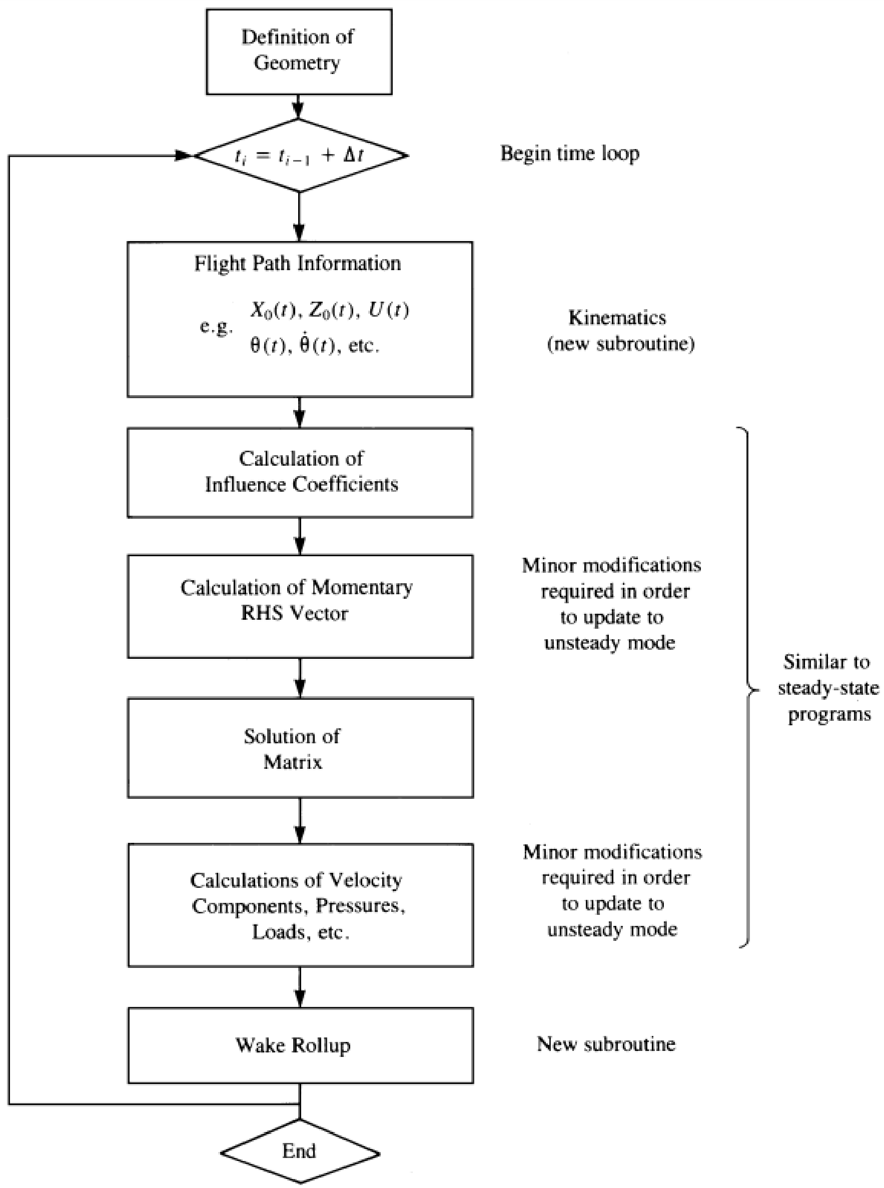
$$RHS_i = \sin(\theta_i - \alpha)$$

$$RHS_i = -[U(t) + u_w, W(t) + w_w]_i \cdot \boldsymbol{n}_i$$

$$\begin{pmatrix} U(t) \\ W(t) \end{pmatrix} = \begin{pmatrix} \cos\theta(t) & -\sin\theta(t) \\ \sin\theta(t) & \cos\theta(t) \end{pmatrix} \begin{pmatrix} -\dot{X}_0 \\ -\dot{Z}_0 \end{pmatrix} + \begin{pmatrix} -\dot{\theta}\eta \\ \dot{\theta}x - \frac{\partial\eta}{\partial t} \end{pmatrix}$$

$$u_w = \frac{\gamma_i (zc_i - zw_i)}{2\pi {r_{ij}}^2}$$

$$w_w = \frac{\gamma_i (xc_i - xw_i)}{2\pi {r_{ij}}^2}$$



[1] Katz, J. and Plotkin, A. (2001)

Modifications for the unsteady state (cont.)

Wake rollup subroutine[1]

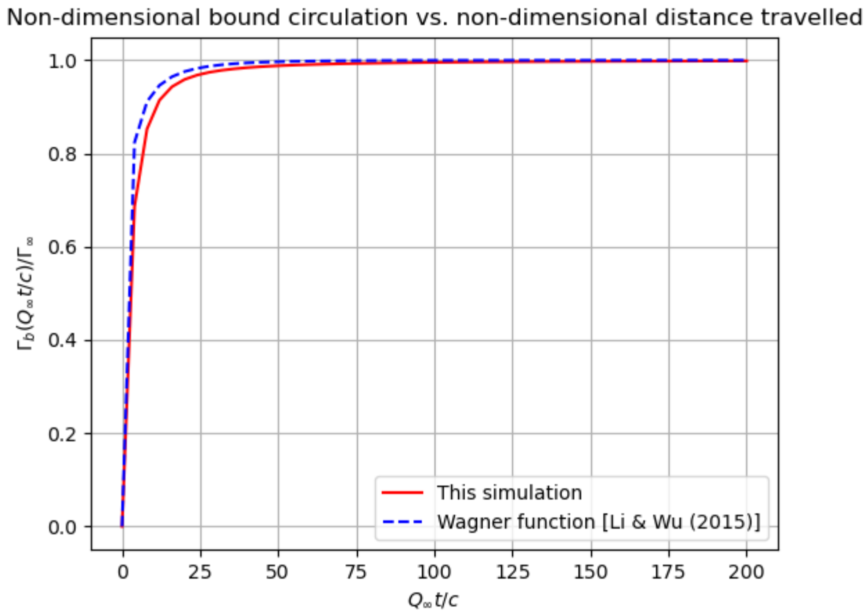
- Ensure that the airfoil's circulation varies with time (implement Kelvin's condition) for unsteady airfoil's wake shedding.
- The local velocity calculated by the velocity components induced by the wake and airfoil. Measured in the inertial frame of reference X, Z.
- At each time step, the induced velocity $(u, w)_i$ at each vortex wake point is calculated, then the vortex elements are moved by

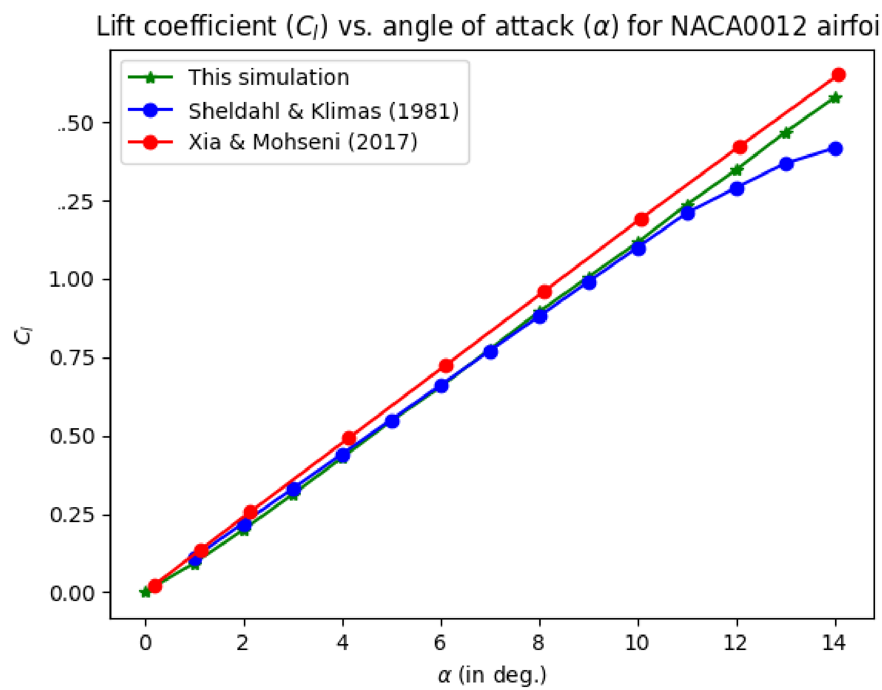
$$(x, z)_i = (u, w)_i \Delta t$$

- Velocity induced at each wake vortex point is a combination of the airfoil and wake vortices.

[1] Katz, J. and Plotkin, A. (2001)

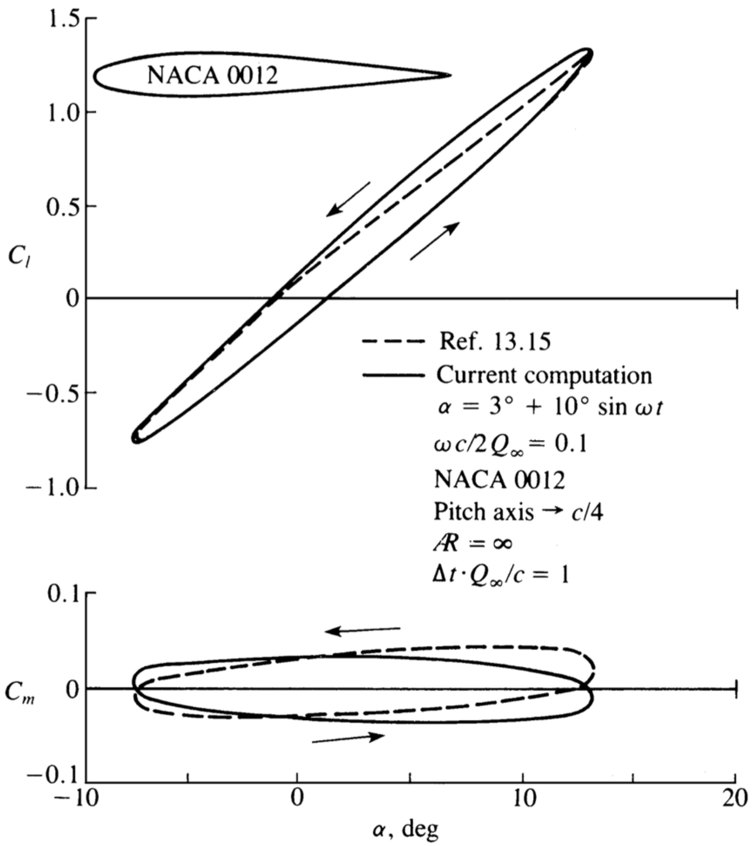
Sudden Forward motion

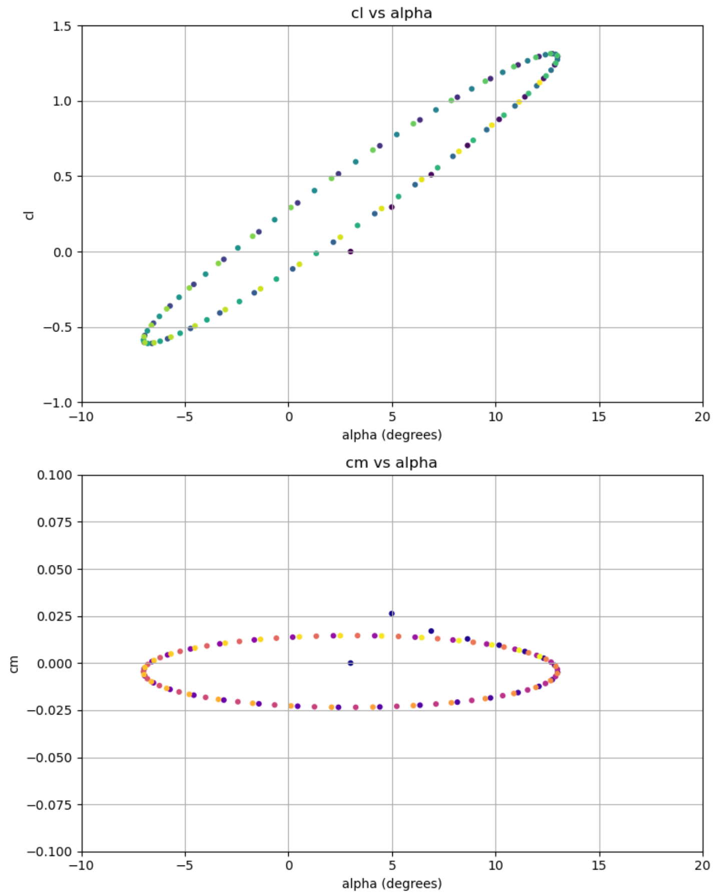




[1] Li and Wu (2015)
[2] Xia and Mohseni (2017)
[3] Sheldahl and Klimas (1981)

Pitching motion

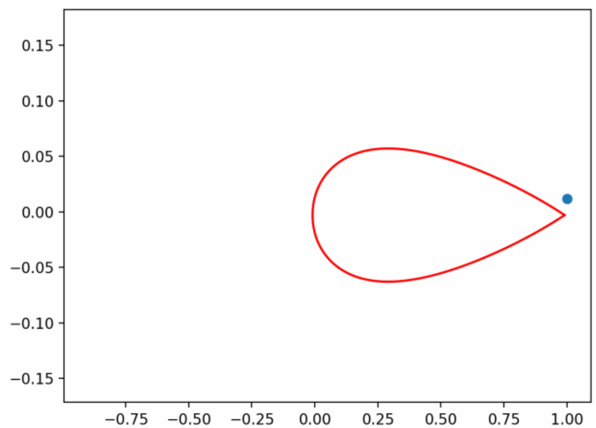




[1] Katz, J. and Plotkin, A. (2001)

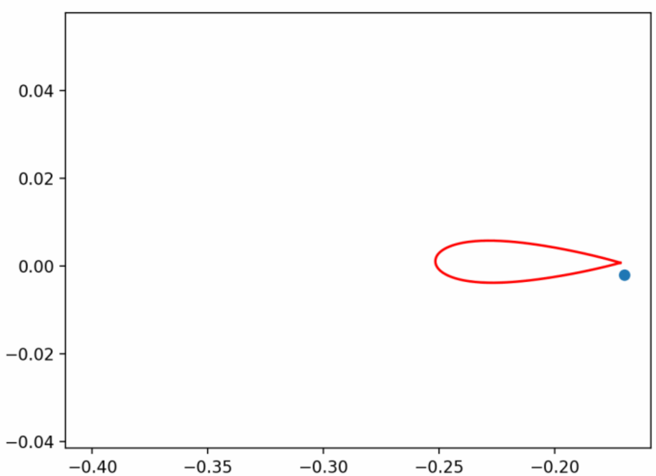
Animations

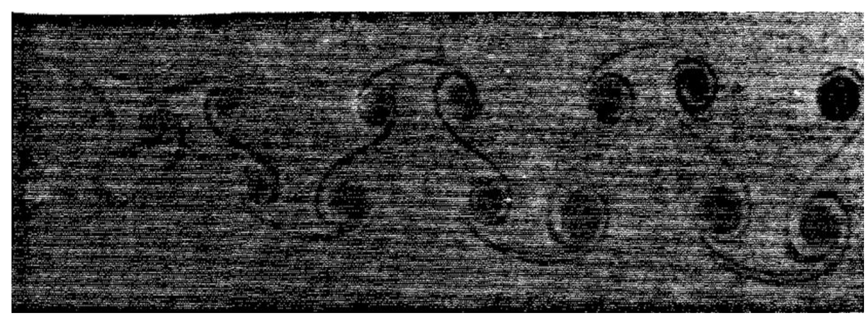
Heaving



$$Q_\infty = 0.9$$
$$\frac{\omega c}{2Q_\infty} = 8.5$$
$$\frac{Q_\infty \Delta t}{c} = 0.009$$

Pitching





[1] Katz, J. and Plotkin, A. (2001)
[2] Koochesfahani (1989)

Future Work

- Attempt to introduce a deformation parameter to realistically model the motion of fish
- Integrating viscous forces and LEV in future simulation framework(s)
- Looking at more complex geometries
- Observe the motion of the fish with the deforming tail in the context of multiple individuals, progressing to looking in the context of a swarm
- Extend model to 3D