**'A literature review on the extent of the influence of mathematical ideas on the natural world'**

## Abstract

It is a common misconception that mathematics is restricted to the modern technological world. This dissertation presents a comprehensive evaluation of spirals, sequences, patterns and behaviours of animals in nature which dictate everyday survival of all the species on the planet along with a brief examination of the theories put forward to explain snowflake structures and river meanders, thus giving a viewpoint of the elegance of the governance of the natural world, both animate and inanimate, through the field of mathematics.

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## Introduction

“Mathematics is to nature as Sherlock Holmes is to evidence.”

-Ian Stewart (Stewart, 1995)

From sand dunes to snowflakes to spider webs, from river meanders to rock formations and beyond, this planet is rife with mathematical accuracies and the power of mathematics is seen in abundance. This dissertation shall endeavour to show only a glimpse of the works of nature that incorporate these patterns in everyday life, whether it is to survive in a desolate landscape, whether these patterns have been formed as a part of an ongoing process, or whether it is to support life as seen with the delightful medley of colours as shown by the butterfly’s wing or the delicate tendrils of the grape vines. Such patterns not only form a vital part of our existence on earth through their ingenuity but also appeal to the human species as a whole through their inherent ability to spot patterns in their surroundings.

It shall be a difficult task to categorise the inherent abilities of animals which have been perfected over millions of years. Take for example, the honeybee’s honeycomb. At first, it seems easy to explain the origins of such complex structures, but considering the fact that humans themselves are good architects, and yet have to resort to methodical analysis of the right material to use, and consider the mathematics behind architecture. This is an unknown gift of architecture that bees possess, which is still unexplained.

There are numerous authors who have also written about such wonders in nature through books like *‘Patterns in Nature’* wherein author Peter Stevens (Stevens, 1974) poses questions as to the structural similarities throughout nature, linking the branching of trees, to that of arteries in the human body, to rivers which he relates to the meandering of cable loops linking the structures of fern fractals to the spirals of galaxies and hurricanes. Thus he manages to show the vast array of mathematical perceptions through examples in nature, by showing their similarities if they were all broken down into the simplest of structural formations. It appears to be the case that the structures in our surroundings all come down to basic rules which define the bigger picture. Thus, it is possible to imagine that mathematics is the tool used by humans the world over, to make sense of these basic rules.

It is clear that we have now begun to escape from nature into the realms of modern, pure mathematics to support our ideas for these natural patterns, but the question as to the purpose behind their existence still holds countless mysteries.

It also led John Adams to note

> 'Another fundamental (and philosophical) question has been asked by many—How can it be that mathematics, a product of human thought independent of experience, is so admirably adapted to the objects of reality?' (Adams, 2003, pp. 3-4)

Thus, this dissertation shall try to explore the influence of mathematics and patterns through logarithmic spirals, fractals and the Fibonacci sequence in nature and where possible, the underlying mathematical concepts behind our extraordinary surroundings and yet it shall do the matter no justice whatsoever as the intricacies and the intuition behind these formations cannot be grasped through one text alone, one can only satisfactorily describe them to the best of their abilities. Does this highlight our failure to understand this mathematical awareness or is it a mark of our ignorance of this universe?

## 1. The Fibonacci sequence and the Golden ratio

The year 1202 AD saw Leonardo of Pisa, more commonly known as Fibonacci, discovering a recurrence relation in sets of rational numbers, and this relation is shown as follows

$$F_n = F_{n-1} + F_{n-2}$$ (Marketos, 2014)

Mathematicians have noticed startling connections between classical geometry and the appearance of this number. The golden rectangle can be constructed in quite a few ways, of which one is mentioned below. A unit square should be constructed, and a line be drawn from one of its vertices to its midpoint. From the same midpoint, construct an arc passing through the vertex adjacent to the one that was used to draw the line. Extend the edge of the square from which the arc was drawn and create a vertex at the point of intersection of the extended edge and the arc. Extend the edges to complete a rectangle as shown below. The ratio of the width to the length of the rectangle is $1{:}\frac{1+\sqrt{5}}{2}$ or $1{:}\varphi$.

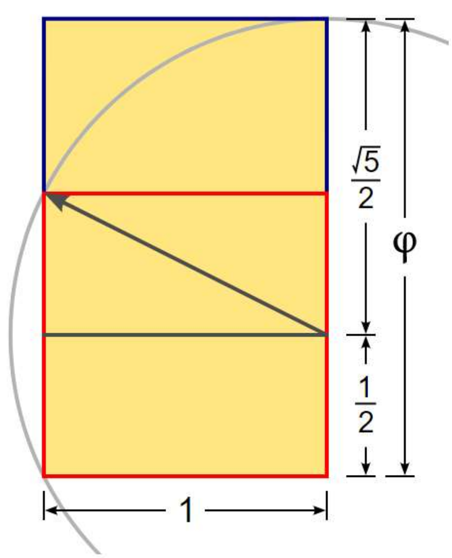


*Figure 1: Construction of a golden rectangle from a square (Holdsworth, 2007)*

Thus the golden ratio is $\frac{1+\sqrt{5}}{2}$ or approximately 1.618, accurate to 4 significant figures. (Weisstein, 2002). This particular number curiously features through the concept of nested radicals and continued fractions, both concepts studied extensively by maths enthusiasts.

Suppose the following nested radical converges when $a \geq 0,\ b \geq 0$ where the limit is $l$.

$$\therefore \sqrt{a+b\sqrt{a+b\sqrt{a+\cdots}}} = l$$

$$\therefore l = \sqrt{a+bl}$$

$$\therefore l^2 - bl - a = 0$$

By the quadratic formula,

$$l = \frac{-b \pm \sqrt{b^2+4a}}{2}$$

Substituting $a = b = 1$

$$\therefore \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}} = \frac{1 \pm \sqrt{5}}{2}, where\ \frac{1+\sqrt{5}}{2} = \varphi$$ (Long, 1967)

Phi ($\varphi$) may also be considered as the numerical value obtained through the iteration of $= \frac{1}{\varphi}$ .

Taking the initial approximation of $\varphi_0 = 1$ , $\varphi_{n+1} = 1 + \frac{1}{\varphi_n}$ for $n \geq 1$ .

So, $\varphi_2 = 1 + \frac{1}{1+\frac{1}{1}}$, $\varphi_3 = 1 + \frac{1}{1+\frac{1}{1+\frac{1}{1}}}$ ... (Rose, 2014). Thus considering $y(\varphi) = x$, the graph below shows the numerical approximation of $\varphi$.

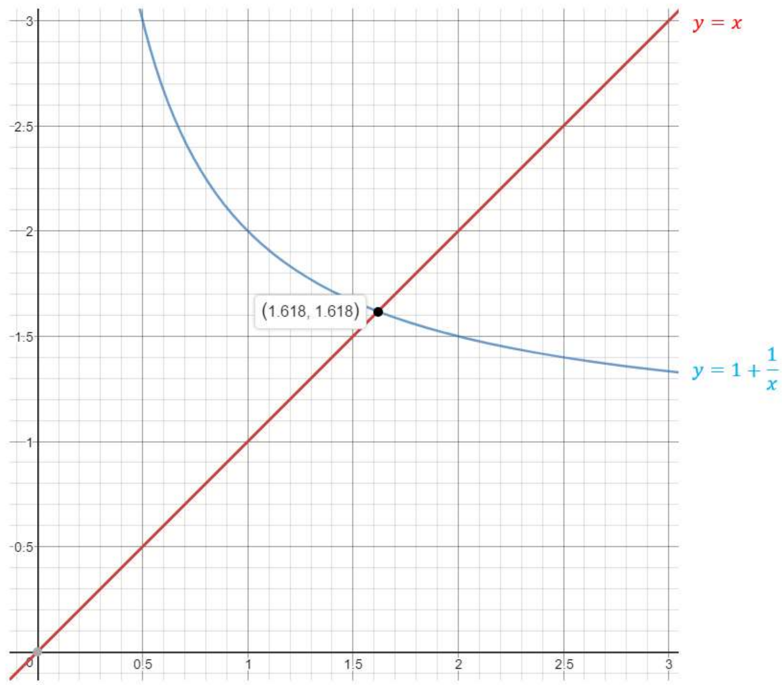


*Figure 2: Graphical representation of φ (Desmos Graphing Calculator, 2015)*

$$\therefore 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}} = \varphi$$

These mathematical quirks have not gone unnoticed by the natural world which has employed its use in efficiently sorting out the perfect arrangement for leaves and seeds to give the maximum output possible for the plant, whether it is in the form of space to grow, or the easy dispersion of seeds or the energy output for the plant (produced due to improved and efficient leaf formations corresponding to the golden ratio) through careful leaf arrangements. The pattern becomes evident through the reproduction of honeybees as well as through a glimpse into the structure of our DNA.

## 1.1 Occurrences of the golden ratio in the plant kingdom

This dissertation shall remain incomplete without discussing the phenomenon of phyllotaxis, which is the arrangement of leaves on the plant stem. When compared to abstract, pure mathematics, it shows some interesting properties. One of them is “the remarkable fact that the numbers of spirals which can be traced through a phyllotactic pattern, are predominantly integers of the Fibonacci sequence” (Erickson, 1983). Indeed, the phenomenon of leaf arrangements is so closely linked with the Fibonacci series, wherein each successive number is the sum of the preceding two numbers; it is astounding that this singularity is ubiquitous in nature. Ravenstein makes note of the fact, that the intersection point of spirals in a sunflower, if coinciding with the position of the leaf, 95% of the time (Ravenstein, 1986, pp. 1-2) follows this recurring Fibonacci sequence. Adler’s hypothesis (Adler, 1974, pp. 1-79) (Adler, 1977, pp. 29-77) to explain this recurring sequence, suggested that the divergence angles in cellular outgrowths in leaves converge to the golden ratio. Ridley (Ridley, 1982, pp. 1-11) managed to verify Adler’s hypothesis through computer simulations, as it produced a Fibonacci phyllotaxis which is closely linked to the golden ratio. One could deduce that this mechanism allows the maximum exposure from sunlight which accentuates the process of photosynthesis in plants whilst the arrangements of the leaves allows any water droplets which settle on them to reach the roots using a mechanical system, which also allows the plant to conserve energy and still allow the plant to receive all the water it possibly can. This is one of the instances of what could be called directed evolution (by mathematics) in organisms.

This relationship between the Fibonacci sequence and the spirals in different directions was observed by Coxeter (Coxeter, 1961). He states that the pineapple shows 8 rows of scales slanting to the left and 13 rows slanting to the right. It is evident that these are Fibonacci integers but astonishingly, the ratio of the two integers approximately equals the golden ratio (1.625). Similarly, the capitulum (centre of a flower) has 34 clockwise and 21 anticlockwise spirals (Prusinkiewicz, 1990). Yet again, both of them are Fibonacci integers and the ratio gets closer and closer to the golden ratio (1.619). A sunflower head has 34 spirals in one direction, and 55 in the other. The ratio of the spirals is 1.6177 (McNabb, 1963, p. 59) which is the golden ratio, correct to three decimal places. It is seen that this arrangement of seeds in the above fashion reduces spaces between the seeds thereby increasing the number of seeds that the plant head can effectively hold thereby increasing the probability of reproduction.

One can say for sure that this shows evolution in action, thus linking plant reactions to a natural stimulus. The next part of the chapter will focus on the mathematics in the animal kingdom.

## 1.2 Occurrences of the golden ratio in the animal kingdom

As previously discussed, the ubiquity of mathematical concepts comes to the fore as the golden ratio occurs in the structure of families in organisms as well as being encoded in the building blocks of life that is the DNA.

### 1.2.1 Reproductive Dynamics

Basin (Basin, 1963, pp. 53-56) describes the genealogical tree of the male bee, which shows the Fibonacci sequence. The male bees (m) have only a single parent as they hatch from an unfertilized egg while female bees (f) have two parents as they hatch from a fertilised egg.

The diagram below by Basin clearly shows the Fibonacci sequence through 7 generations of bees.

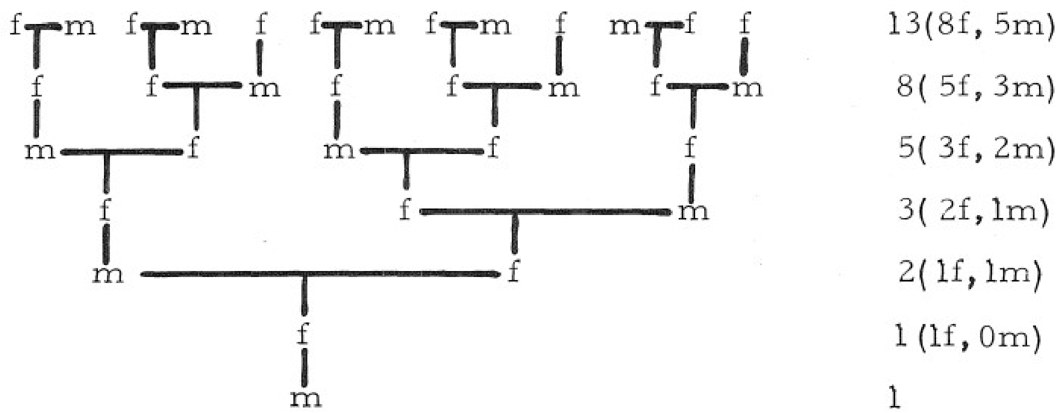


*Figure 3: Familial structure of bees and the Fibonacci sequence (Basin, 1963, p. 53)*

### 1.2.2 Structural DNA

There has been constant research into internal relationships between spirals of the DNA and the search for patterns within these sequences and research by Wahl recognises that the DNA is created out of two pentagons rotated by 36° creating a golden decagon with their sides in the ratio of $1\!:\varphi$. He further notes that the molecule measures 34 by 21 angstroms and the ratio of their length to width is approximately 1.619 (Wahl, 1988). This shows only a 0.04% difference from the golden ratio.

Furthermore Perez (Perez, 2009) has revealed that there is a certain relationship between codons (a sequence of three combinations of the 4 bases in our DNA, combining to form a unit of DNA (Nature.com, 2017)) and the golden ratio ($\varphi$) and his results showed that 3 parameters (1, 2, and φ) define codon populations within 20 different species of animals to a precision of 99% and often 99.999%. The precision in his experimental data shows clear signs of a relationship between $\varphi$ and DNA sequences in animals, thus inspiring future explorations into mathematical genomics to understand the structure of humans from a mathematical standpoint.

## 1.3 Occurrences of the golden ratio in the inanimate world

Surprisingly, the golden ratio is seen not only in our surroundings, but also in the behaviour of atoms, which make up our universe. Considering a compound of the magnetic material, cobalt niobate, Coldea (Coldea et al., 2010) managed to show that the linked magnetic atoms in cobalt niobate, when encountered with a magnetic field on the atom chains would cause resonance in the atoms themselves (the tension from the interactions between spins of atoms cause them to magnetically resonate), which cause them to interact through a certain ratio that is 1.618. Coldea further states that 'it reflects a beautiful property of the quantum system' and that it shows 'a hidden symmetry.'

This shows the perceptions of leading experts in their fields about the astonishing and unexplainable patterns seen in the natural world.

Thus, mathematicians and biologists see the golden ratio as a tool alike as a foray into understanding the intricate workings of nature. It is evidence to the fact that these patterns are constant irrespective of size and nature of the elements involved. However, the existence of mathematics in nature is not just limited to certain ratios or proportions, but it is inbuilt into the lives of each individual, in every species on the planet and to some extent the inanimate world as well.

## 2. The inherent mathematical abilities and properties in organisms

One may compare mathematics with artistic beauty in nature, as has been the primary intention throughout this paper. Plato once said that the beauty of style, harmony and rhythm depend on simplicity (Nguyen, 2015). Thus, this paper focuses on both the simple and challenging concepts required to understand the world around us and wishes to emphasize the fact that mathematical ideas encompass the natural world. It comes to the fore through the instinctive processes of organisms, and through millions of years of evolution, which have let creatures like bees build architecturally sound structures.

It seems odd to think that each organism is programmed in a particular fashion, just so that it may exist in this world dependent on its base programming or *instinct*. Bees themselves possess such abilities as aforementioned. But when the origins of the idea that bees are able to comprehend numerical cognition, comes into question, it is a puzzle to understand the bees' innate ability to determine the right thickness and width of the honeycomb whilst creating a sturdy structure, and has led scientists to question whether it is a result of certain regulatory principles or genetic predisposition.

Yet it is clear today why bees have chosen the hexagonal tessellations over countless other Euclidean shapes, for the simple reason that the hexagons leave very little space between tessellations, require the minimum amount of wax for construction and yet can store the maximum amount of honey, thus being quite resourceful. Although the cell walls are only 0.05 mm thick, they can support 25 times their weight (Murthy, 2013, pp. 6-8). This still does not explain how the bees inherently understand such complex geometric perceptions or how it could be a product of millions of years of evolution.

This suggests that animals are inherently programmed to understand mathematics at a basic level, which is evident through the study of numerical cognition in animals. Numerical cognition is the ability to understand and act upon a given problem through logic. These patterns are also evident through spiral structures, which are incorporated by animals to support life.

## 2.1 Numerical cognition in animals

The variety in thought processes and reflexes in organisms apart from humans, insinuate that they are capable or arranging their thoughts and help model how they experience the world. This advantage has been attributed to humanity alone, but this is certainly not the case. Original research (Cantlon, 2007) shows that rhesus monkeys solved 40 arithmetic problems with a mean accuracy of 76% as compared to college students with mean 96% accuracy. Yet, the mean response time for the monkeys was 1099 $ms$ as compared to 940 $ms$ for humans (Cantlon, 2005) (Cantlon, 2006). This suggests that humans and monkeys have a similar procedure to approaching and solving arithmetic problems based on the relative similarities of their mean response times.

Chimpanzees are considered quite similar to monkeys with humans rhesus monkeys and chimpanzees sharing 93% of the same DNA (Arbanas, 2007) thus it is unsurprising that studies on Ai, a female chimpanzee has shown that Arabic numerals can be used to represent numbers, and can order 9 elements in increasing order of magnitude and studies also showed that Ai could remember sets of information with 5 elements, which is the same as pre-school children whereas as adult humans can remember numerical information up to 7 elements (Kawai, 2000).

This further provides evidence to the hypothesis that humans and other species of monkeys and chimpanzees have similar basic numerical cognitive abilities, which stem from certain algorithms which are as yet unexplained. Further studies on bonobos and apes could further this study in numerical reasoning abilities in primates.

## 2.2 Spirals and their representation in nature:

Spirals are generally described as structures, which have an increasing radius of curvature (Thompson, 1945). i.e. two structures following the same spiral structure which may or may not be mathematically similar, as they grow only in one direction.

Bernoulli's spiral, or the logarithmic spiral is ubiquitous in nature and is described (Pickover, 1988) as

$$r = ke^{a\theta}$$

where $r$ is the distance from the origin, $\theta$ is the angle between the straight line and the tangent to the curve, which is constant, while $k$ and $a$ are arbitrary constants and $e$ is Euler's number ≈ 2.718 to 3 decimal places (Shell-Gelasch, 2008). This is only one of the hundreds of times that this number appears whilst trying to understand the natural world, but these circumstances cannot be completely discussed in this dissertation, due to the magnitude of the occurrences.

Spirals in nature are ubiquitous in nature and have a range of zoological and botanical manifestations (Kawaguchi, 1982). The most common of these are the logarithmic spirals on the nautilus shells and the horns on the African kudu antelope (Haeckel, 1998). Martin Gardener has noted the existence of logarithmic spirals in the webs through running strands coiling outwards from the centre in Eperia, which is a common genus of spiders (Gardner, 1969). Such spirals can also be created through algorithms, which simulate the structure of invertebrates through polynomial equations alone (Pickover, 1987).

It is clear to me that the ubiquity of patterns in nature is attributed to the presence of spiral structures in the natural world, which form the basis for spots and stripes on animal fur, as well as the variety of patterns on seashells. The types and formations of such patterns shall be discussed further and mathematical models shall be used to understand these phenomena.

## 3. Some mathematical models used to describe patterns in nature

Humans pride themselves on developing logical arguments to support their hypothesis that nature can be modelled by manipulating mathematical laws, so it is possible that the very characteristics that define an animal could be explained through critical mathematical theories.

Even stripes and spots on animal fur, are no mean feats of evolution, at least not completely. In the mid-20th century Alan Turing (Turing, 1952) published a paper describing a theory for the production of stripes and spots through a reaction-diffusion model of mathematics, which compelled the production of such characteristics. He describes a simple model, including an 'activator', and an 'inhibitor' for morphogens, which are chemical substances which induce responses dependant on concentration of the chemical substances (Tabata, 2004), which diffuse (particles spread from an area of high concentration to an area of low concentration) through tissue (a substrate) and then react within it, thus creating a reaction-diffusion process. Consider two morphogens, X and Y, where X is a morphogen having the ability to create black hair, while Y is a morphogen having the ability to create white hair in an organism. Thus in regions where X is abundant, the animal will have only have black hair, while regions dominated by Y will have white hair. Supposing X can stimulate production of other X molecules along with acting as a catalyst for reactions for Y, thus becoming an *activator*, (a catalyst being a substance facilitating reactions between substances but not actually taking part in the reaction itself (Davis, 2003) but subdues the formation of Y (Y is an *inhibitor*)). Thus in places with high concentrations of X, there will be high concentrations of X *and* Y, but if Y diffuses faster than the reactions with X, Y will spread out in a small area, till such a point where the concentration of X is reduced, thus forming specific darker areas amongst plainer backgrounds as showed by dappled patterns calculated by Turing.

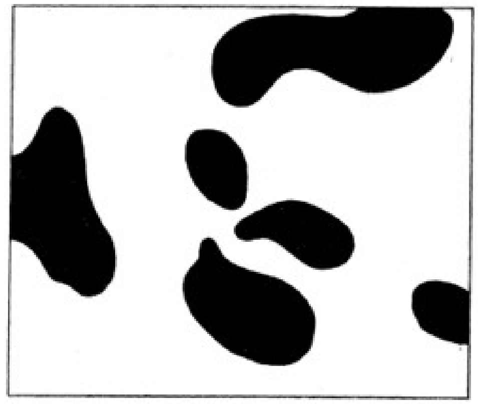

*Figure 4: Dappled pattern from a reaction-diffusion system (Turing, 1952, p. 60)*

These patterns closely resemble patterns on pandas and Dalmatians, and can be slightly modified to resemble stripes on zebras. Thus, supposing Turing's theory is correct, considering white sheep, that particular Y morphogen is in abundance with little to no concentrations of that particular X morphogen and vice-versa in black sheep. Recent investigation into this model has justified Turing's theory to be correct (Castets et al., 1990, pp. 2953-2956) (Tompkins et al., 2014, pp. 4397-4402).

Fowler et al. (Fowler, 1992) worked on a refined model of Turing's theory and described two partial differential equations as follows,

$$\frac{\partial a}{\partial t} = \rho s\left(\frac{a^2}{1+\kappa a^2}+\rho_0\right) - \mu a + D_a \frac{\partial^2 a}{\partial x^2} \qquad \dots (1)$$

$$\frac{\partial s}{\partial t} = \sigma - \rho s\left(\frac{a^2}{1+\kappa a^2}+\rho_0\right) - \nu s + D_s \frac{\partial^2 s}{\partial x^2} \qquad \dots (2)$$

Fowler et al. describe $\alpha$ as the concentration of the activator at the rate $D_a$ which decays at the rate $\mu$ . Thus the partial differential equation (1) shows the change of concentration of the activator with respect to change in time. The substrate (eg. tissues in the skin/fur) has a concentration of $s$ and decays at the rate $\nu$ . Constant rate of production of the substrate involved is $\sigma$ and the constant of proportionality between $\sigma$ and $\alpha^2$ is $\rho$. The parameter $\kappa$ represents the variable of saturation of the activator concentrations in the substrate. The partial differential equation (2) shows the change of the concentration of the substrate with respect to change in time. This helps us understand the sort of parameters to be considered whilst deriving these equations as well as giving a glimpse into the variables which influence such formations on animal bodies.

Varying values for the above parameters have shown to give stunning, almost lifelike images for seashell patterns. Fowler et al. draw parallels with Sabelli's (Sabelli, 1979) seashell entries as shown below by certain variations of the aforementioned parameters. Yet, discussing the origins or the nature of Fowler et al.'s equations is beyond the scope of this paper.

*Figure 5: Volutoconus bednalli (L) and a superimposed approximation through Fowler et al.'s (1992) equations (R)* $\rho = 0.1 \pm 2.5\%, \rho_0 = 0{:}\,0025, \mu = 0.1, D_a = 0.01, \sigma_{max} = 0{:}\,11, \upsilon = 0{:}\,002, D_s\ 0{:}\,05, and\ \kappa = 0.5$

*Figure 6: Amoria undulata (L) and a superimposed approximation through Fowler et al.'s (1992) equations (R)* $\rho = 0.1 \pm 2.5\%, \rho_0 = 0.005, \mu = 0.1, D_a = 0.004, \sigma_{max} = 0.012, \upsilon = 0, D_s = 0, and\ \kappa = 1.$

Thus mathematical models help explain mathematical systems better and are able to make predictions based on the various parameters involved. Similarly, the natural world self-improvises, linking to the development of self-learning and self-improving algorithms. Thus the human species takes inspiration from the natural world, but the extent of the same is beyond the limit of this dissertation. This mathematical modelling extends even to the inanimate forms of nature including the formation of snowflakes and we shall also be talking in brief about the meandering paths of rivers, which relates to our original idea of a natural world being completely governed by mathematical laws, whether animate or inanimate.

## 4. Geometry and fractal theory in nature

### 4.1 Snowflakes and fractal theory

The term 'fractal' was coined by Mandelbrot (Mandelbrot, 1982, pp. 170-171) which is a repeating pattern that retains its original shape and structure when magnified. It is recognised as a subset of the set of lines as compared to being part of Euclidean geometry (Shenker, 1994, pp. 968-969). i.e. the geometry comprised from Euclid's axioms (the point, lines, segments, etc.). Yet, Herge von Koch, in 1904 (Weisstein, 2008) showed that through an iterative formula, one could transform a Euclidean shape to a more complex structure through fractal theory. This "well-worn out pattern" as described by Mandelbrot (Mandelbrot, 1982, pp. 174-175), to explain Kepler's ideas of motion of planets and comets in orbit, is truly astonishing. Koch managed to show the complex and unique nature of snowflakes forming in the atmosphere, as shown below.

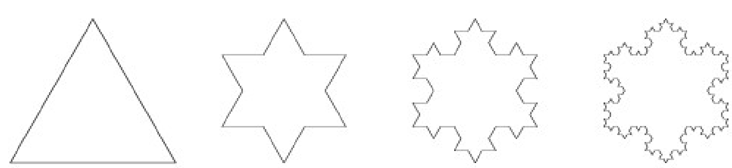

*Figure 7: Koch's snowflake formations (Weisstein, 2008)*

Libbrecht (Hoffman, 2011) suggests that as snowflakes are formed through a mix of certain temperatures and a certain wind flow, these patterns are unique as no two snowflakes go through the exact same phase, there would be many different variations and possibilities – even exceeding the number of atoms in the universe, according to Libbrecht. Exactly how these formations came to be, is as yet not fully understood.

Koch's snowflakes suggest that, any iteration of triangles with triangles rotated through *n* degrees would produce unique patterns in snowflakes, as seen below (Weisstein, 2008). What is interesting is the fact that the von Koch snowflake curve has an infinite perimeter and any two points on the perimeter are an infinite distance apart, even if the area of the snowflake itself is finite.

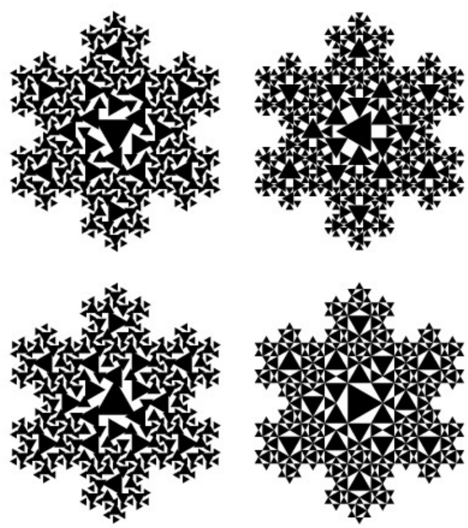

*Figure 8: Rotational snowflake formation (Rouvray, 1996, p. 83)*

It is clear that such formations exhibit 'self-similarity' (Flook, 1996) i.e. if the formation is magnified to show a smaller part of the bigger version, they will be similar to each other. Such is the power of fractals, as they can be created through computer-generated algorithms;

the question therefore, is whether these patterns are a mere coincidence or whether it is a structural development throughout nature.

## 4.2 Meanders: Geometry of rivers

Realising the need for a generalised theory to explain the winding 'threadlike patterns' of river flow patterns, Lazarus & Constantine (2013) have set up and explained a couple of parameters which may influence the path that the route including the roughness of the landscape as well as the gradient of the river bed relative to each other.

Similarly, Langbein & Leopold (1966) examined the sinuosity of the Colorado River in Utah, USA and have come up with sine curves to model the behaviour of the pattern in which the river meanders. They have suggested that there is a constant value to the ratio of the sinuosity of the river to its radius of curvature which is approximately $4.7:1$. They also explain a random walk model which suggests that the ability of the river to curve (the sinuosity) has a finite probability to deviate by some angle $\theta$. This suggests that the river follows the most probable form that the river can take.

As Stevens has mentioned before, the meanders of rivers are indeed like cable loops, but their existence is not entirely in disorder according to Stølum's research (Stølum, 1996). His research shows that the sinuosity has a mean of 3.14 (3 s.f.) or approximately π, with values ranging from 2.7 to 3.5, according to realistic computer simulations and observational and experimental data.

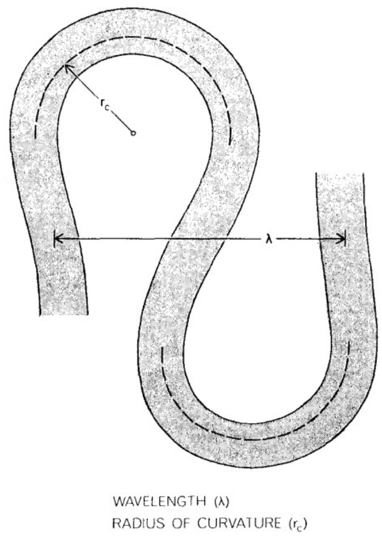


*Figure 9: Approximation of the curvature of a river (Leopold, 1966)*

This concludes our brief assessment of the curvature of rivers as well as the appearance of fractal theory in snowflakes.

## 5. A study to determine the influence of mathematical sequences and concepts on the human subconscious

Humans too form part of the natural world and thus are linked to nature via the mathematical laws that have been discussed in brief during this dissertation. Yet, when the inherent mathematical skills in animals were discussed, the inherent skills in humans were overlooked. Thus, the study is now a foray into the human psyche. This is a proposition of a study to determine how the mathematical laws intrinsically govern the way humans think, which is based on the original experiment designed by Gustav Fechner (Benjafield, 1985; Boselie, 1992) in 1876. The hypothesis is that there is a positive correlation between the frequencies of selection of rectangles which show the golden ratio, wherein the chosen rectangle is chosen intuitively, to show a subconscious operation of mathematics in the human body.

## 5.1 Methodology

A type of non-probabilistic sample i.e. a convenience sample of size 15 was chosen, because as a student it was difficult to gain access to a wider sample, due to a lack of contacts, in order to be able to generalize the results. The findings were calculated based on an original questionnaire (Appendix A). The experiment involved creating eight rectangles with different length to width ratios, with only the third rectangle type with a ratio close to the golden ratio. A preliminary test based on the same questionnaire was carried out in our institution, wherein certain flaws were found and corrected and these shall be explained in more detail. The method of collecting data, was to hand out the questionnaires around the neighbourhood, and the participants only selected one rectangle that they perceived to be the most pleasing to their subconscious. The idea of *most pleasing*, is defined to be when the length and width of the rectangles are in an inherently agreeable aesthetic proportion pertaining to that individual. To define aesthetics, one must now conform to traditional ideas of psychology i.e. the study of design and the environment through human experiences and perceptions. As design is a human phenomenon, so aesthetics is by definition a psychological progression. It is this aesthetic proportion that is being investigated in this subjective experiment. The most pleasing rectangle according to my hypothesis was to be the third rectangle. Considering that a centimeter rule was used to measure the dimensions, the ratio of the third rectangle is $1.627 \pm 0.045\ (3\ d.p)$. As the golden ratio lies within this range, this rectangle was considered useful for the experiment. To this end, the fourth responder believed that the third rectangle was the most pleasing, which does agree with the original hypothesis. But when the whole sample itself is considered, only 20% of the respondents replied that the most pleasing rectangle was the third one.

## 5.2 Discussion

The research conducted here is qualitative research and this idea is backed by certain facts including an emergent design in the research process, along with using theoretical sampling techniques to obtain data for the research. Accordingly, as no statistical analysis can be performed on my results and that the repeatability of my questionnaire is impossible, it stands to reason that this research is not quantitative.

The preliminary test showed that none of the 10 participants agreed that the most pleasing rectangle was the third rectangle. This was attributed to a high percentage uncertainty in the value for the golden ratio, which was used for the third rectangle. With certain changes to the length-width ratio of the third rectangle, the final questionnaire succeeded in showing that 20% of the participants agreed that the third rectangle was the most pleasing.

These results are the data based upon which all subsequent exploration will take place. Thus, the outcome of the experiment could be explained by the lack of a greater sample size, as one cannot project the result obtained onto the wider population. These results may also have turned out the way they did, as a result of the research process. Fechner used a multiple choice system as opposed to a single choice system. As Fechner's results determined that only 35% of the respondents chose the golden rectangle amongst the others presented, which increased to 41% after considering the adjacent rectangles chosen by the participants, 24% still chose rectangles with ratios $\leq 1.45$ or $\geq 2.00$ (McManus et al., 2010). Thus, it is suggestive of the fact that this decline in results is primarily due to a change in methodology used as he incorporated a 'like' and 'dislike' judgement made by each participant, and thus worked with a range of values, instead choosing a single rectangle was better for the study as ranking the rectangles in order would have forced the participants to start to recognise patterns in how the rectangles were formed or the order in which they were arranged in. This was crucial to avoid, as the hypothesis clearly states that an instinctive response to the selection of rectangles is required to obtain an objective and unbiased viewpoint of the human subconscious.

The data from the present sources leave the study of preferences of rectangles in humans in a predicament as there is no substantial evidence or a model to prove or disprove the hypothesis, and it is clear that more work on the methodology used as well as the choice of participants is required.

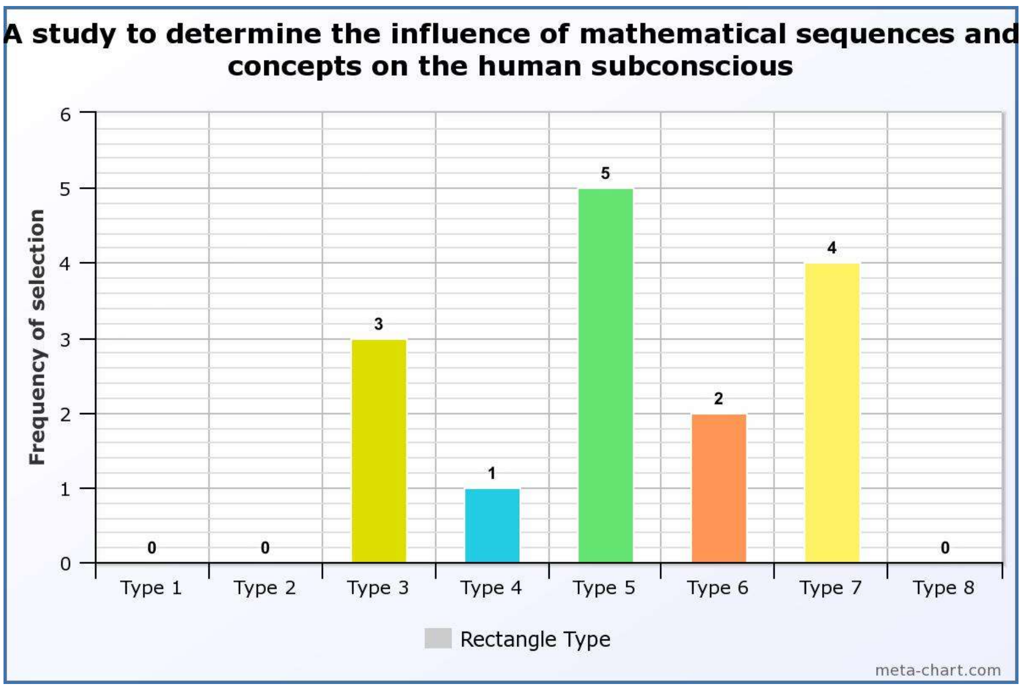


*Figure 10: Findings from the study to determine the influence of mathematical sequences and concepts on the human subconscious (Meta-chart, 2017)*

## 5.3 Limitations

My study is primarily restricted by my views on objectivity, that is to say, a human's ability to have inclinations towards designs cannot be limited to one or two shapes, and rather a system where the top three preferred rectangles were chosen would be more focused than either of the methods used by Fechner, or myself.

The study was also constrained by the fact that an equal ratio of males and females were not implemented during the data collection process. This constraint stems from the idea that males and females perceive and have different viewpoints regarding designs and art.

The participants were also not segregated on the basis of their age or mental abilities (schizophrenia) (Green et al., 2004) or other factors such as their extraversion in society, or their Holland types (Doer, Thinker, Organiser etc.) which shows their individuality (Holland, 1959) which can influence how aesthetics is perceived in such individuals. Therefore, a questionnaire should have been designed keeping in mind such demographics.

## Conclusion

Mathematical concepts have been found to encompass and suggest a sense of equilibrium to the natural world and do not seem to differentiate based on size, whether they are inanimate or animate or whether they belong to the plant or animal kingdom. This suggests an interdependent relationship between the mathematical models humans have come up with and the natural phenomena seen around us. More specifically, it is clear that animals integrate certain algorithms which help their numerical perception; it is a generalised view that is being made, as chimpanzees and monkeys, with a close genetic description to humans were considered. A future study could be conducted on the basis of this sub-point to try and evaluate various arithmetic skills in a variety of primates, or could be broadened to include other animals as well to provide a comprehensive evaluation by testing mammals and aves (birds) separately and documenting their reactions to the mathematical stimulus. Thus this foray into several aspects of mathematical biology asks us whether or not, these mathematical appearances can be chalked down to happenstances or whether there is a treasure trove of knowledge of the natural world that we have yet to unlock. Yet, this literature review is supplemented by the results of the original experiment which does not provide empirical evidence to support the idea that human mathematical responses to stimuli can be tested through their perceptions of the golden ratio through geometrical shapes. It is clear that Fechner's approach (with respect to my approach) can be considered inadequate to study an archetypical case of rectangle predispositions in human beings.

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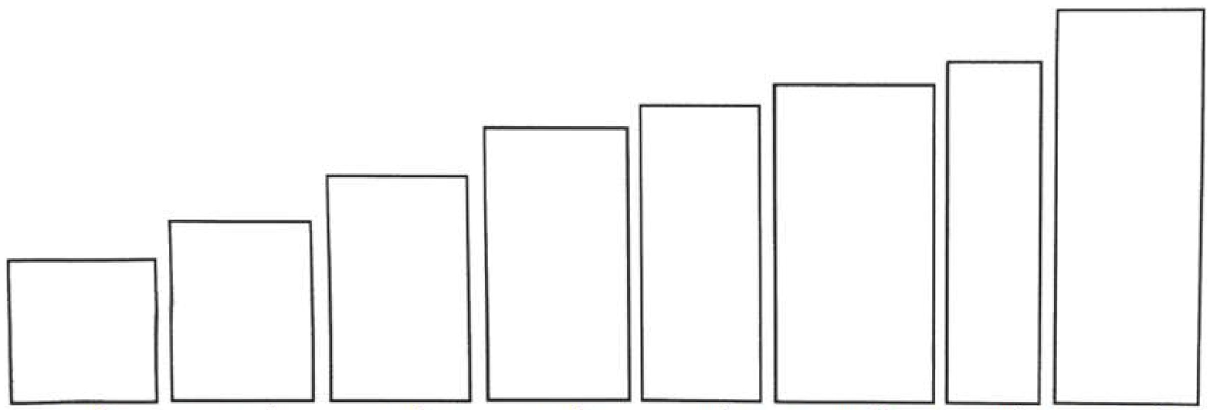
## Appendix A

A study to determine the influence of mathematical sequences and concepts on the human subconscious *19 May 2017*

*Select a single rectangle that you feel is the most aesthetically pleasing and insinctively appeals to you.*
*Put a tick mark in the selected rectangle.*



Rectangle 1 Rectangle 2 Rectangle 3 Rectangle 4 Rectangle 5 Rectangle 6 Rectangle 7 Rectangle 8

*Not to scale